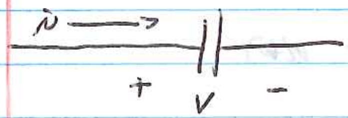
EL 2013 - Fall 2014 ~~Rec~~ Recitation 8 - 10/28/2014

Active vs. Passive elements. Passive elements cannot supply energy! Active elements can supply energy.

Capacitance is defined by a voltage - current relationship.

$i = C\frac{dv}{dt}$ Capacitance is measured in Farads

$1F = \frac{1 A\cdot s}{V} = \frac{1C}{V}$



Charge is stored on a conducting plate. Charge accumulates on the surface of the metal plate and does not enter interior

A capacitor is 2 conducting plates w/ area A separated by distance d and has capacitance $C = \varepsilon A/d$

ε = permittivity (insulating material)

For air $\varepsilon = \varepsilon_0 = 8.85 pF/m$

$\boxed{\text{Practice 7.1}}$ $C = 5mF$

a) $v = -20V$ $i = C\frac{dv}{dt} = (5\times10^{-3}) \times -20\frac{dv}{dt}$

$= (5\times10^{-3})(0) = 0A$

b) $v = 2e^{-5t} V$ $\therefore I = C\frac{dv}{dt} = (5\times10^{-3})(-5\times 2e^{-5t})$

$= -50\times10^{-3} A = \boxed{-50mA = I}$

Current - Voltage relationship: $i = C\frac{dv}{dt} \Rightarrow dv = \frac{1}{C} i(t)dt$

Integrate between times t + t_0

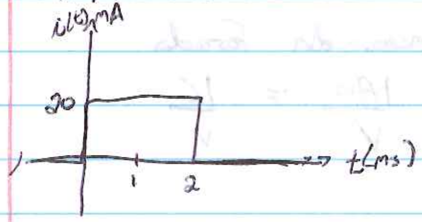
$$\int_{v(t_0)}^{v(t)} 1 dv = \frac{1}{C}\int_{t_0}^{t} i(t')dt'$$

$$v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(t')dt' \quad \therefore \quad v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')dt' + v(t_0)$$

In many cases, $t_0 = -\infty$ + $V(-\infty) = 0$, such that

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt'$$

Example 7.2



$C = 5\mu F$

$$v(t) = \int_{t}^{t_0} \frac{1}{C} i\,dt' + v(t_0)$$

Interval

$-\infty \le t \le 0$ $\quad$ $i = 0 \to V = 0$

$0 \le t \le 2ms \to$ $\quad v(t) = \int_{0}^{2ms} \frac{1}{C}\, i\,dt' + v(0)$

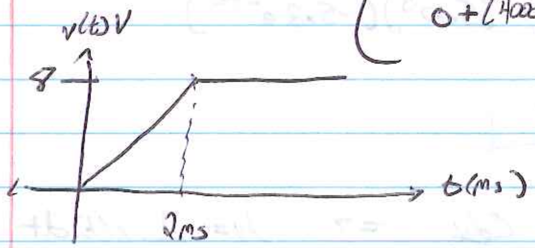
↳ = 4000t

$2ms \le t \le \infty \to$ ↳ $0 + v(2ms)$

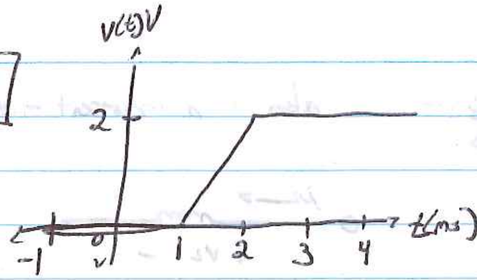
$$= \frac{1}{C}\int_{0}^{t} 20\times10^{-3}\,dt' + 0$$

since current is constant $\quad = \frac{1}{(5\times10^{-6})} \times t\,20\times10^{-3}\Big|_{0}^{t} = 4000t = v(t)$

$$\therefore\ v(t) = \begin{cases} 0 & \text{for } -\infty \le t \le 0 \\ 4000t & \text{for } 0 \le t \le 2ms \\ 0 + (4000)(2ms) = 8V & \text{for } 2ms \le t \le \infty \end{cases}$$



Practice 7.2



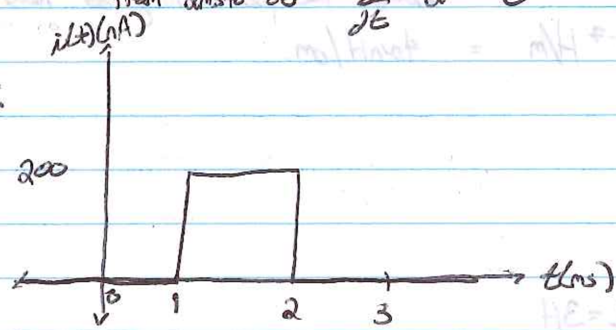
$C = 100pF$

$i = C\frac{dv}{dt}$ Interval

$-\infty$ to 1ms $\qquad \frac{\partial}{\partial t}0 = 0$

1ms to 2ms $\qquad 100pF \frac{d}{dt}\frac{2t}{1\times10^{-3}} = 200\times10^{-9}A = 200nA$

From 2ms to ∞ $\qquad \frac{d}{dt}2 = 0$



Energy storage $\qquad P = V \cdot I = Cv\frac{dv}{dt}$

$\int$ Power = Energy $\qquad P = Cv\frac{dv}{dt}$

$$\int_{t_0}^{t} p\,dt' = C\int_{t_0}^{t} v\frac{dv}{dt'}\,dt'$$

$$P = \frac{dE}{dt} = C\frac{dv}{dt} \rightarrow dE = C\,dv\frac{dt}{dt}$$

$$\int_{t_0}^{t} dE = C\int_{v(t_0)}^{v(t)} dv \rightarrow E_c(t) - E_c(t_0) = C\left[\frac{v(t)^2}{2} - \frac{v(t_0)^2}{2}\right]$$

$$\therefore E_c(t) - E_c(t_0) = \frac{1}{2}C\left[v(t)^2 - v(t_0)^2\right]$$

if $E(t_0) = 0$ then $E_c(t) = \frac{1}{2}C\,v(t)^2 = \frac{1}{2}Cv^2$

Inductor $v = L\frac{di}{dt}$ also a current - voltage relationship

$1H = 1\frac{V \cdot s}{A}$

$i_L \rightarrow$

$+ \; v_L \; -$

winding wire into a coil. # of turns affects the current

in general $L = \mu N^2 \frac{A}{s}$ where N = # of complete turns of wire

A = cross sectional area

s = axial length of ~~the~~ the helix

μ = the permeability of helix material

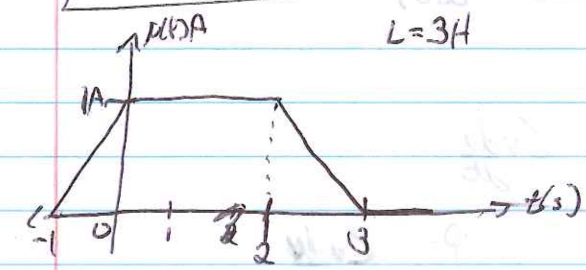
~~air~~ in air

$\mu = \mu_0 = 4\pi \times 10^{-7}\ H/m = 4\pi\ nH/cm.$

$v = L\frac{di}{dt}$

Example 7.4

$L = 3H$



Interval

$-\infty$ to -1 current = 0, $V = 0$

-1 to 0 $L \cdot \frac{di}{dt} = L \cdot 1 t = 3 \cdot 1 t \frac{d}{dt}$

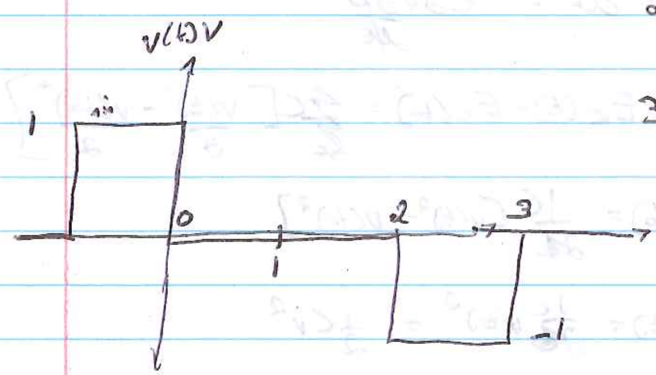
$= 3V$

0 to 2 $3 \cdot 1 \frac{di}{dt} = 0$ constant current

= constant voltage.

2 to 3 $\rightarrow$ $3 \times \frac{-t\,d}{dt} = -3V$

3 to ∞ $\rightarrow = 0$ constant current = 0V



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Practice 7.4

$L = 200mH$

$v_L = L\frac{di}{dt}$

$\frac{di}{dt}$ is the rate of change of the current which is the derivative of the current or slope of the line over an interval

slope $= \frac{rise}{run}$ $\frac{\text{change in } y}{\text{Change in } x}$

For Figure 7.13

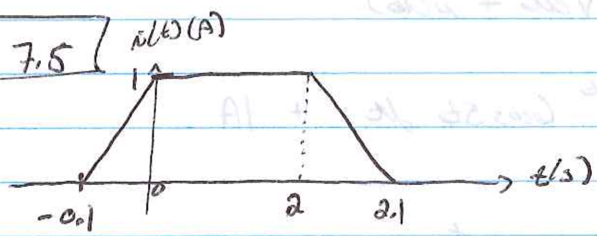
Interval		Voltage
$-\infty$ to $-3ms$	no current	$V = 0V$
$-3ms$ to $1ms$	$= 200mH \times \frac{2}{1} t \frac{d}{dt}$	$= 200\times10^{-3} \times 2 = 400mV$
$1ms$ to $3ms$	$= 200mH \times \frac{2}{2} t \frac{d}{dt}$	$= 200\times10^{-3} \times 1 = 200mV$
$3ms$ to $6ms$	$= 200mH \times \frac{4}{3} t\,dt = -\frac{800}{3} mV$	$= -266.67mV$

a) $v_L(0) = 0.4V$

b) $v_L(2ms) = 0.2V$

c) $v_L(6ms) = 0.266V$

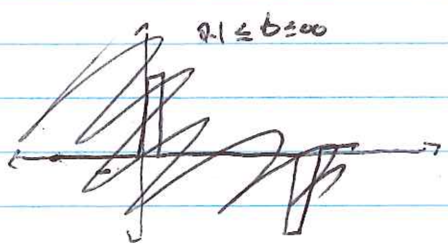
Example 7.5

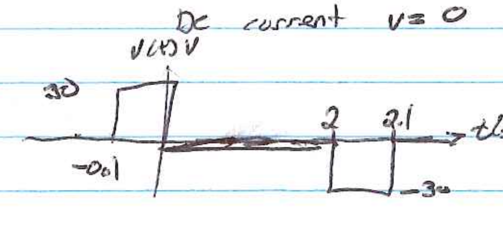


$L = 3H$

$V = L\frac{di}{dt}$

Interval	Voltage
$-\infty \le t \le -0.1$	DC current → $V = 0$
$-0.1 \le t \le 0$	$\frac{d}{dt}\frac{1}{0-0.1} = 10 \times 3H = 30V$
$0 \le t \le 2$	DC current → $V = 0$
$2 \le t \le 2.1$	$3H \cdot \frac{d}{dt} = 3 \times \frac{-1}{2.1-2} = 3 \times -10 = -30V$
$2.1 \le t \le \infty$	DC current $V = 0$





Integral relationship

$$v = L\frac{di}{dt}.$$

$$di = \frac{1}{L} v\,dt \qquad \int_{i(t_0)}^{i(t)} i\,di = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

$$= \quad i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

$$\rightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt' + i(t_0)$$

$$\therefore \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)$$

Example 7.6 $\quad L = 2H \qquad v = 6\cos 5t$

if $i(t = -\pi/2) = 1A$ $\qquad$ make $t_0 = -\frac{\pi}{2}$ $\quad i(t_0) = 1A$

$$\therefore \; i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$= \frac{1}{2} \times \int_{t_0}^{t} 6\cos 5t\,dt + 1A$$

$$\therefore \; \frac{1}{2}\,\frac{6}{5}\sin(5t)\Big|_{t_0}^{t} + 1A$$

$$= \frac{1}{2}\left[\frac{6}{5}\sin(5t) - \frac{6}{5}\sin\left(5 \times -\frac{\pi}{2}\right)\right] + 1A$$

$$= \frac{3}{5}\sin(5t) - -\frac{3}{5} + 1 \rightarrow \boxed{i(t) = \frac{3}{5}\sin(5t) + 1.6\ A}$$

Practice 7.6 $L = 100mH$ $v_L = 2e^{-3t}V$

$i_L(-0.5) = 1A$ $t_0 = -0.5$ $i(t_0) = 1A$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} 2e^{-3t}\,dt + i(t_0)$$

$$i(t) = \frac{1}{100\times10^{-3}} \times \frac{-2}{3}e^{-3t}\Big|_{t_0}^{t} + 1A$$

$$\therefore i(t) = \frac{20}{3}\left[e^{-3t}\Big|_{-0.5}^{t}\right] + 1A$$

$$= \frac{20}{3}\left[e^{1.5} - e^{-3t}\right] = \frac{20}{3}e^{-3t} + 30.8779\,A = i(t)$$

Energy storage $p = v \cdot i = Li\frac{di}{dt}$

$$p = \frac{dE}{dt} = Li\frac{di}{dt} \qquad dE = L\,i\,di$$

$$\int_{t_0}^{t} dE = L\int_{i(t_0)}^{i(t)} i\,di \rightarrow E_L(t) - E_L(t_0) = L\left[\frac{i(t)^2}{2} - \frac{i(t_0)^2}{2}\right]$$

$$\therefore E_L(t) - E_L(t_0) = \frac{L}{2}\left[i(t)^2 - i(t_0)^2\right]$$

if $E_L(t_0) = 0$, $E_L(t) = \frac{1}{2}Li^2$

Example if $C = 12pF$

$+ v_C(2) = 10V$, $v_C(0) = 4V$, $w_C(0) = 2nJ$

what is $w_C(t)$ at $t = 2s$

Eq. 3 page 222, $w_C(t) - w_C(t_0) = \frac{1}{2}C[v(t)^2 - v(t_0)^2]$

$$\therefore w_C(t) = \frac{1}{2}C[v(t)^2 - v(t_0)^2] + w_C(t_0)$$

$$= \frac{1}{2}(12\times10^{-12})[(10^2) - (4^2)] + 2nJ$$

$$\boxed{= 2.504\,nJ}$$