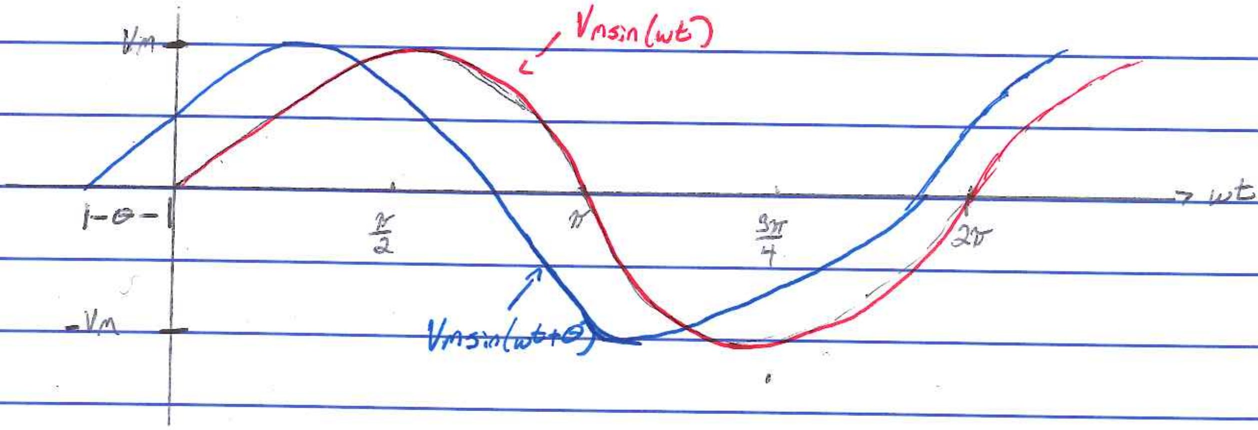
Recitation 13 - 12/2/2014



we can say that $V_m\sin(\omega t)$ lags $V_m\sin(\omega t+\theta)$ by θ radians

or that $V_m\sin(\omega t)$ leads $V_m\sin(\omega t+\theta)$ by $-\theta$ radians

These sinusoids are out of phase, but sinusoids with the same phase are in phase with eachother

Sines and Cosines are out of phase by 90°

In general $\sin\omega t = \cos(\omega t - 90°)$

or $\sin(\omega t + 90°) = \cos(\omega t)$

In this case the sine leads the cosine.

Practice 10.1 $v_1 = 120\cos(120\pi t - 40°)V$

a) $i_1 = 2.5\cos(120\pi t + 20°)A$ $\quad$ i_1 lags by $-40-20=-60°$

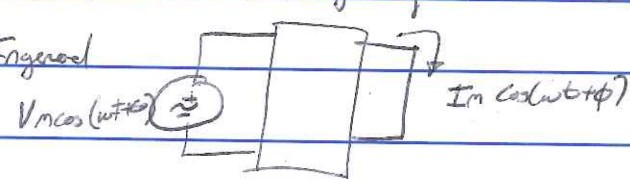
b) $i_2 = 1.4\sin(120\pi t - 70°)A = 1.4\cos(120\pi t - 160°)$; i_1 lags by $-40-160 = 120°$

c) $i_3 = -0.8\cos(120\pi t - 110°)A \Rightarrow$ ~~i lags by~~ $\rightarrow 0.8\cos(120\pi t + 70°)$ i_1 lags by $-40-110 = 70°$

$\boxed{\text{Identities}}$ $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$V_m\cos(\omega t + \theta - 90°) = V_m\sin(\omega t + \theta)$

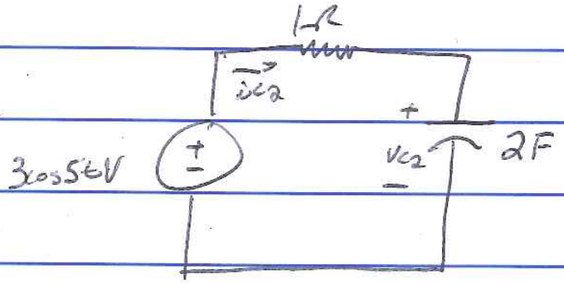
In general



Imaginary Sources lead to imaginary responses!

Imaginary Source $\rightarrow V_m\cos(\omega t+\theta) + jV_m\sin(\omega t+\theta) \rightarrow V_m e^{j(\omega t+\theta)}$

Example 10.2



$i = C\frac{dv}{dt}$

By Definition $V_s = i_{c2} \times 1 + v_{c2} = 0$ $\qquad i_{c2} = \frac{C\,dv_{c2}}{dt}$

$\therefore 3e^{j5t} = 2 \times \frac{\partial v_{c2}}{dt} + v_{c2}$

$\therefore -3e^{j5t} + 2\frac{\partial v_{c2}}{dt} + v_{c2} = 0$

at steady steady $v_{c2} = V_m e^{j5t}$ in general Form

$\therefore -3e^{j5t} + 2 \times V_m e^{j5t}\frac{d}{dt} + V_m e^{j5t} = 0$

$-3e^{j5t} + 10j\, V_m e^{j5t} + V_m e^{j5t} = 0$

$V_m e^{j5t}(10j + 1) = 3e^{j5t}$

$V_m = \frac{3}{(10j+1)} \quad || = \underbrace{\frac{3}{\sqrt{10^2+1^2}}}_{mag} \cdot \underbrace{\angle -\tan^{-1}(10/1)}_{phase} V$

$\therefore V_m = \cancel{\frac{3}{\sqrt{1+10}}} \cdot \frac{3}{\sqrt{101}} \angle -\tan(10/1) V$

where $Re\{V_{c2}\} = Re\{V_m e^{j5t}\} \to Re\left\{\frac{3}{\sqrt{101}} \angle \tan^{-1}(10/1)\, e^{j5t}\right\}$

$\boxed{= 0.2985\cos(5t - 84.3°)V = v_{c2}}$

Phasor Form

IF $v(t) = V_m \cos(\omega t)$, $V = V_m \angle \theta = V_m \angle 0$

IF $i(t) = I_m \cos(\omega t + \phi)$, $I = I_m \angle \phi$

Appendix 5 helps w/ complex #'s + Euler's identity

Still holds true that $V = RI$

$v(t) = 100 \cos(400t - 30°)$ V

$V = 100 \angle -30°$ V

Inductor $L \rightarrow V = j\omega L I$ From $V = L\frac{di}{dt}$

Capacitor $C \rightarrow I = j\omega C V$ From $I = C\frac{dv}{dt}$

$\cancel{\#}$ From $V \rightarrow V_m e^{j(\omega t + \theta)}$

$I \rightarrow I_m e^{j(\omega t + \phi)}$

$$V_m e^{j(\omega t + \theta)} = L I_m e^{j(\omega t + \phi)} \frac{d}{dt}$$

$$= V_m \cancel{e^{j\omega t}} e^{j\theta} = Lj\omega I_m \cancel{e^{j\omega t}} e^{j\phi} \rightarrow V_m e^{j\theta} = Lj\omega I_m e^{j\phi} \therefore \boxed{V = Lj\omega I}$$

Example 10.4

Apply $8\angle -50°$V at a frequency of $\omega = 100$ rad/s to a 4H inductor determine phasor current + time-domain.

By Definition $I = \frac{V}{j\omega L}$ $\frac{8\angle -50° V}{j(100)(4)} = \frac{8\angle -50°}{400\angle 90°} \rightarrow \frac{8}{400}\angle -140°$

$\therefore I = 0.02 \angle -140°$ A

Impedance + Admittance $\frac{V}{I} = R$; $\frac{V}{I} = j\omega L$; $\frac{V}{I} = \frac{1}{j\omega C}$

Admittance is opposite of resistance.