Recitation 12 - 11/25/2014

Complete response of RLC circuits is similar to RL + RC circuits. Forced responses are produced from DC sources that are switched on or off in the circuit, such that the response may not vanish at $t = \infty$

Steps

① Determine initial conditions

② Obtain numerical value for the forced response

③ Write appropriate form of the natural response with the necessary number of arbitrary constants

④ Add forced response + natural response to make complete response

⑤ Evaluate response and its derivative at $t = \emptyset$ and employ initial conditions to solve for unknown constants

In general for voltage, forced response $v_f(t) = V_F$

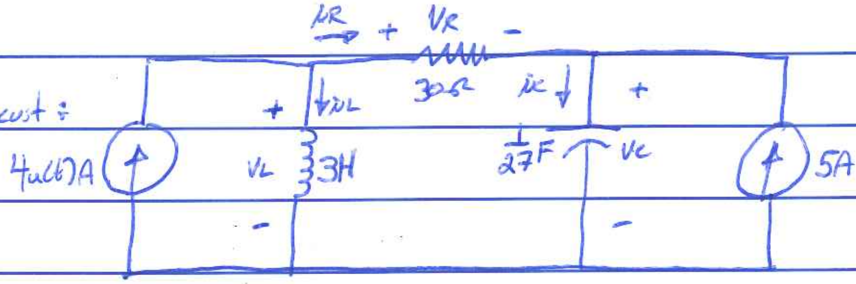
natural response $v_n(t) = Ae^{s_1 t} + Be^{s_2 t}$

$\therefore v(t) = V_F + Ae^{s_1 t} + Be^{s_2 t}$

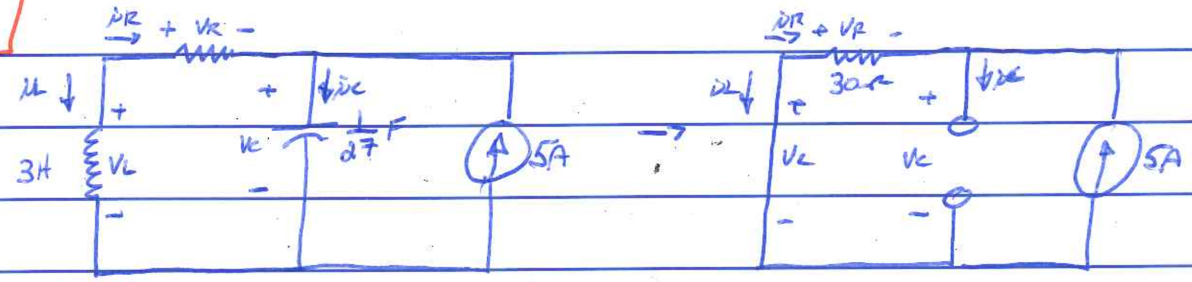
more difficult portion is derivative of response $\frac{dv}{dt} = 0 + s_1 Ae^{s_1 t} + s_2 Be^{s_2 t}$

with $\frac{dv}{dt}$ at $t = 0^+$

Example 9.9   Circuit:



$t = 0^-$



① $i_C(0^-) = 0\ A$ ② $i_R(0^-) = -5A$ ③ $i_L(0^-) = 5A$

④ $v_C(0^-) = 150V = 5\times30$ ⑤ $v_R(0^-) = -150V = -5\times30$ ⑥ $v_L(0^-) = 0V$

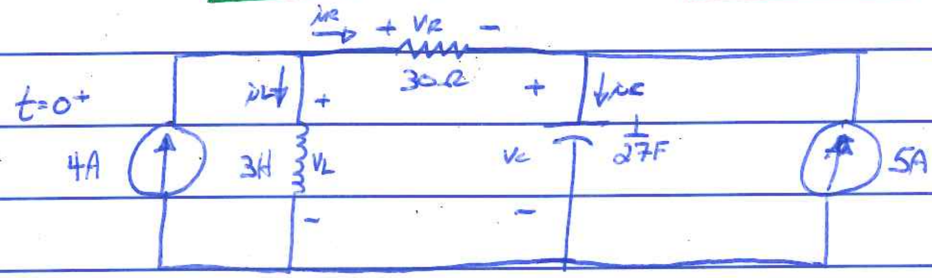
For $t=0^+$ what cannot change?

$\omega_0 = \frac{1}{\sqrt{3\times\frac{1}{27}}} = 3 = \omega_0$

$v_C(0^-) = v_C(0^+) = 150V$ ④ $i_L(0^-) = i_L(0^+) = 5A$ ③

Series: $\frac{R}{2L} = \frac{30}{6} = 5 = a$ $a > \omega_0$

$t = 0^+$



$s_{1,2} = -a \pm \sqrt{a^2 - \omega_0^2}$

$s_1 = -1$ $s_2 = -2$

i_L can't change instantaneously, so $i_R(0^+) = i_R(0^-) + 4A = i_R(0^+) = -1A$ ②

∴ $V_R(0^+) = 30\times\cancel{5} = -30V$ ⑤ $i_C(0^+) = i_R(0^+) + 5 = -1+5 = 4A = i_C(0^+)$ ①

$V_L(0^+) = V_R(0^+) + v_C(0^+) = -30 + 150 = 120V = V_L(0^+) = 120V$ ⑥

Forced response of C is 150V, $v_C(t) = 150 + Ae^{-1t} + Be^{-9t}$

$v_C(0^+) = 150$ ∴ $150 = 150 + A + B$ ∴ $A + B = 0$

$\frac{dv_C}{dt} = -Ae^{-t} - 9Be^{-9t}$ $i_C = C\frac{dv_C}{dt}$ $i_C(0^+) = 4 = C\frac{dv_C}{dt}$

∴ $\frac{4}{C} = -Ae^{-t} - 9Be^{-9t}\Big|_{t=0}$ → $\frac{4}{1/27} = -A - 9B$ $B = 108 + 9B$

$108 = -A - 9B$ $-A = 108 + 9B$ ∴ $B = -13.5$

$A = 13.5$

∴ $v_C(t) = 150 + 13.5(e^{-t} - e^{-9t})\ V$

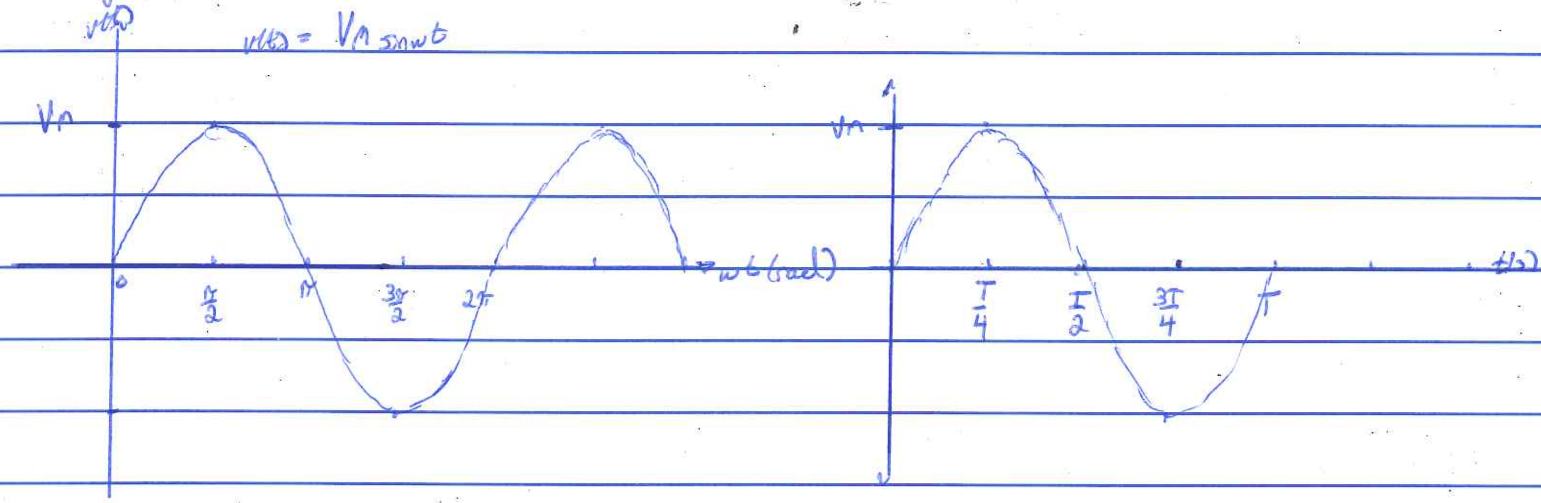
Sinusoids : sinusoidally varying voltage $v(t) = V_m \sin(\omega t)$

V_m = amplitude

ωt = argument where t is time,

ω is radian or angular frequency

may be plotted as a function of t or ωt

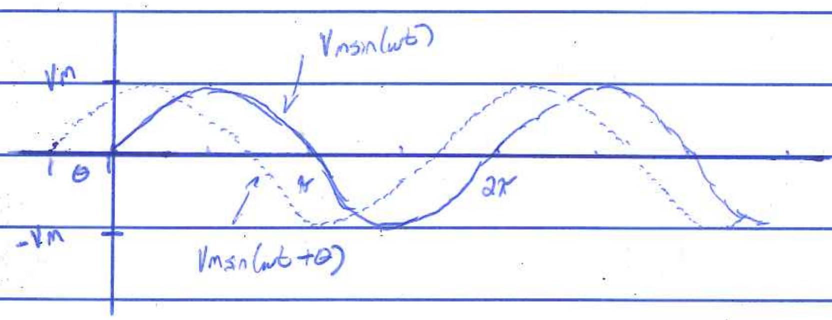


Period is 2π radians, it's periodic every 2π

as a function of t, period is T, executes ~~every~~ $\frac{1}{T}$ periods each second.

$f = \frac{1}{T}$ $\omega T = 2\pi$ $\therefore$ $\omega = 2\pi f$

Lagging + leading



Sines to cosines is just 90° phase shift

Phasors : $e^{jx} = \cos x + j\sin x$

3V DC source $= 3e^{j0°} = 3V$

can represent everything in terms of $e^{j\omega t}$

if $v(t) = V_m \cos\omega t = V_m \cos(\omega t + 0°) \rightarrow V_m \angle 0°$ (polar) complex

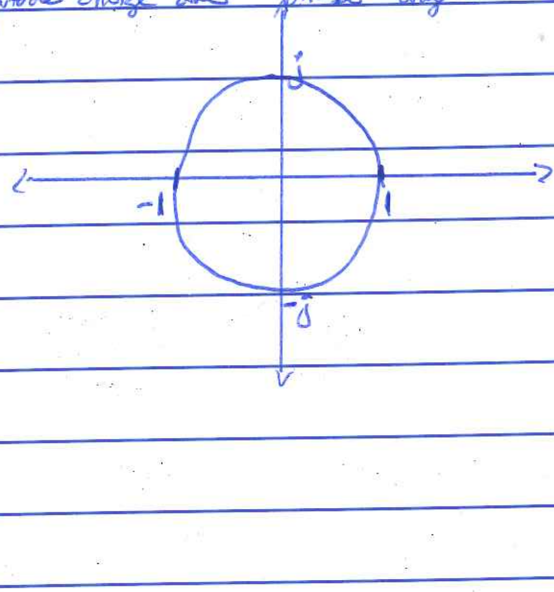
can represent current as $i(t) = I_m \cos(\omega t + \phi) = I_m \angle\phi$ phasor

Steps for turning in to a phasor if $i(t) = I_m \cos(\omega t + \phi)$, it is expressed as the real part of Euler's identity $\rightarrow i(t) = Re\{I_m e^{j(\omega t + \phi)}\}$

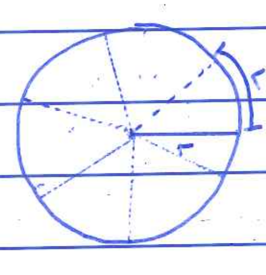
can drop $e^{j\omega t}$ + simply represent $I = I_m e^{j\phi}$, in polar $I_m \angle\phi = I$

Process
1. $i(t) = I_m \cos(\omega t + \phi)$
2. $i(t) = Re\{I_m e^{j(\omega t + \phi)}\}$
3. $I = I_m e^{j\phi}$
4. $I = I_m \angle\phi \rightarrow$ frequency domain representation

magnitude change over phasor angle



Radians



circumference is

2π radians

We can represent everything as phasors!

~~[illegible]~~

Appendix 5 Complex #'s
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General Expression $e^{j\theta} = \cos\theta + j\sin\theta$

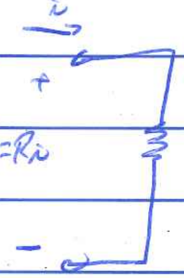
$$Ce^{j\theta} = C\cos\theta + Cj\sin\theta$$

in general $\cancel{V_m}(\cos\theta + j\sin\theta)V_m \rightarrow V_m e^{j\theta}$

$v(t) = Ri(t) \rightarrow$ $v(t) = V_m e^{j(\omega t+\theta)} = V_m\cos(\omega t+\theta) + jV_m\sin(\omega t+\theta)$

$$i(t) = I_m e^{j(\omega t+\phi)} = I_m\cos(\omega t+\phi) + jI_m\sin(\omega t+\phi)$$

$$\therefore V_m e^{j(\omega t+\theta)} = Ri(t) = RI_m e^{j(\omega t+\phi)}$$



$$V_m e^{j\theta} = RI_m e^{j\phi}$$

$$V_m\angle\theta = RI_m\angle\phi$$

Inductor $v(t) = L\frac{di}{dt}$ in phasor form $\rightarrow$

① $V_m e^{j(\omega t+\theta)} = L\frac{d}{dt} I_m e^{j(\omega t+\phi)}$

② $V_m e^{j(\omega t+\theta)} = j\omega L I_m e^{j(\omega t+\phi)}$

③ $V_m e^{j\theta} = j\omega L I_m e^{j\phi}$ $\therefore \boxed{V = j\omega L I}$

Capacitor $i(t) = C \frac{dv(t)}{dt}$

① $\cancel{I_m e^{jt}}$ $I_m e^{j(\omega t+\phi)} = C \frac{d}{dt} V_m e^{j(\omega t+\theta)}$

② $I_m e^{j(\omega t+\phi)} = j\omega C V_m e^{j(\omega t+\theta)}$

③ $I_m e^{j\phi} = j\omega C V_m e^{j\theta}$

④ $\therefore$ $I = j\omega C V$

In general For $R \to$ $V = RI$

$L \to$ $V = j\omega L I$

$C \to$ $V = \frac{1}{j\omega C} I$