Recitation 11 11/18/2014

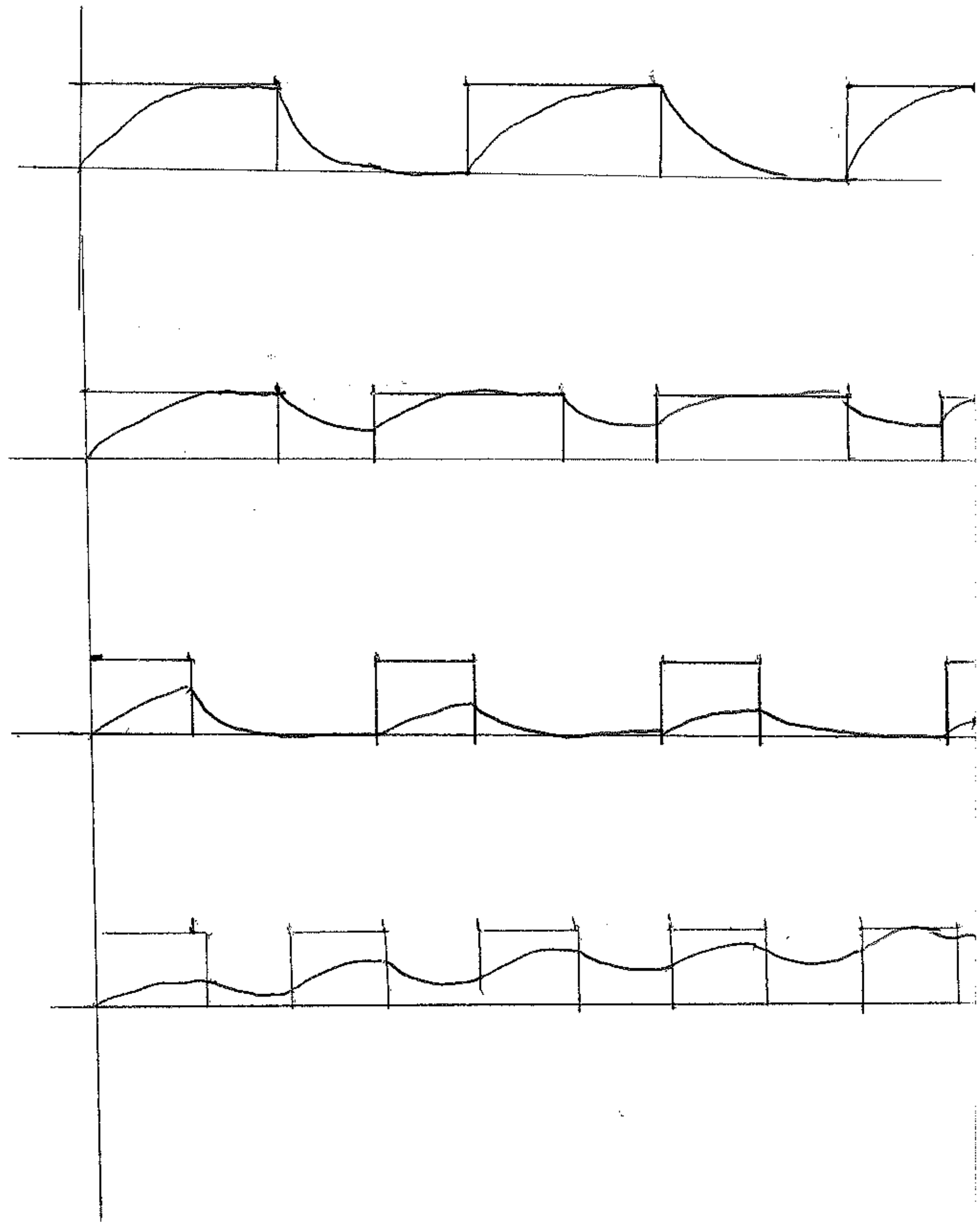
Page 302-304, Charging + Discharging properties of Capacitors + Inductors.

Case I: Fully Charge + Discharge

Case II: Fully Charge but not Discharge

Case III: Fully Discharge, not charge

Case IV: No time to fully charge or discharge;



Two solutions to Differential equation for RLC circuits

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

general Solution for voltage: $V = Ae^{st}$ Substitute $V_1 = A_1e^{s_1t}$

$V_2 = A_2e^{s_2t}$

Linearity proves ~~both~~ sum of 2 solutions is a solution $\therefore$ $V(t) = A_1e^{s_1t} + A_2e^{s_2t}$

For $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ — $s_1 + s_2$ are dimensionless have units defined per second.

so $\frac{1}{2RC}$ + $\frac{1}{\sqrt{LC}}$ also have s^{-1} units. $\frac{1}{s}$ = 1Hz is Frequency.

$\underline{\omega_0}$ = Fundamental or <u>resonant Frequency</u> = $\frac{1}{\sqrt{LC}}$

$\frac{1}{2RC}$ is the <u>neper Frequency</u>, or exponential damping factor, $\alpha = \frac{1}{2RC}$

Rate at which natural response decays or damps to steady state.

s, s_1, s_2 are complex frequencies.

So, for $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$\alpha = \frac{1}{2RC}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$A_1 + A_2$ are found by solving initial conditions

Relationship	s values	Description
$\alpha > \omega_0$	$s_1 + s_2$ are real	over-damped
$\alpha < \omega_0$	$s_1 + s_2$ are complex	underdamped
$\alpha = \omega_0$	real value, just α	critically damped.

$\boxed{\text{Example 9.1}}$ $L = 10mH$, $C = 100\mu F$

Find R ranges for over + under damped.

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\qquad \alpha = \frac{1}{2RC}$

Overdamped: $\frac{1}{2RC} > \frac{1}{\sqrt{LC}} \rightarrow \frac{\sqrt{LC}}{2C} > R$ $\quad \sqrt{\frac{(10\times10^{-3})(10\times10^{-6})}{2(10\times10^{-6})}} > R$ $\quad R < 5$

underdamped $R > 5$

Overdamped RLC circuit $v(t) \to A_1 e^{s_1 t} \to 0$ as $t \to \infty$

$$v(0) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_1 + A_2 = v(0)$$

then use ~~i~~ initial conditions on capacitor

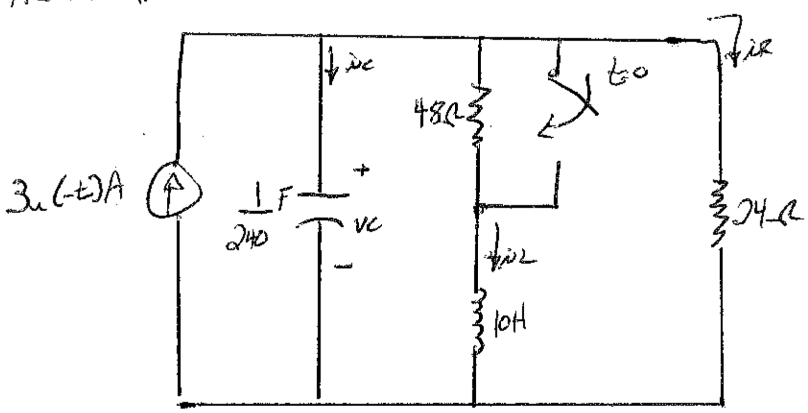
$$i_c = C\frac{dv}{dt} \qquad \frac{dv}{dt} = s_1 A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t}$$

evaluate at $t=0$ $\left.\frac{dv}{dt}\right|_{t=0} \to s_1 A_1 + s_2 A_2$

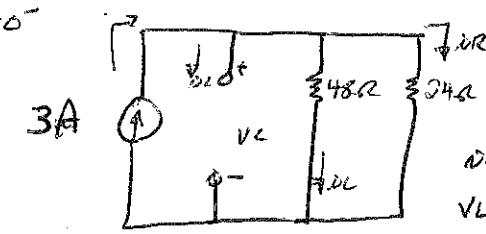
$$\left.\frac{dv}{dt}\right|_{t=0} = \frac{i_c(0)}{C}$$ use this and to solve for $A_1 + A_2$

Use table ~~on page~~ 9.1
on page 347 Response equations are given.

Practice 9.2



$t=0^-$



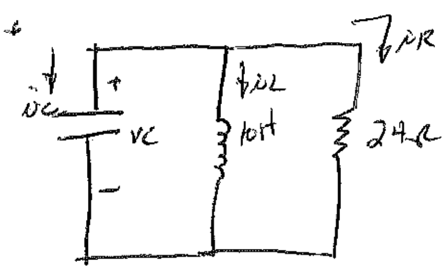
$i_C(0^-) = 0A$
$V_L(0^-) = 0V$

a) $i_L(0^-) = \frac{3 \times 24}{24+48} = 1A$

b) $v_C(0^-) = 3 \times (48 \| 24) = 3 \times \frac{(48 \times 24)}{48+24} = 48V$

$i_L(0^-) = i_L(0^+) = 1A$

$t=0^+$



c) $i_R(0^+) = 3 - i_L(0^+) = 3 - 1 = \boxed{2A = i_R(0^+)}$

d) $i_C(0^+) = -i_L(0^+) - i_R(0^+) = -3A$

e) $V_C(0.2)$

$$\alpha = \frac{1}{2RC} = 5 s^{-1}$$

$$w_0 = \frac{1}{\sqrt{LC}} = 4.89 \text{ rad/sec}$$

$$i_C(t) = C\frac{dv_C}{dt} = C\left[A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}\right]$$

$\alpha > w_0 \to$ overdamped.

$A_1 e^{s_1 t} + A_2 e^{s_2 t}$

use $v_C(0^+) = A_1 + A_2 = 48V$ solve rest