Recitation 10 11/11/2014

Capacitors store voltage. Voltage across a capacitor cannot change instantaneously. If same DC source for a very long time, capacitor becomes open circuit + has open circuit voltage.

Current through a capacitor can change instantaneously (displacement current).

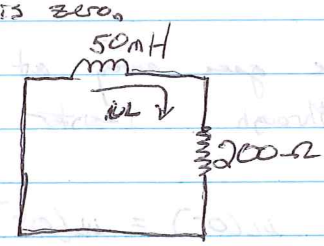
$$I = C\frac{dv}{dt}$$ if voltage is constant, current flow is zero.

Inductors store current. (become current sources) Current through an inductor cannot change instantaneously. If same DC current source is on for a long time, inductor becomes a short circuit current source.

Voltage across an inductor can change instantaneously

$$V = L\frac{di}{dt}$$ if current is constant, voltage potential is zero.

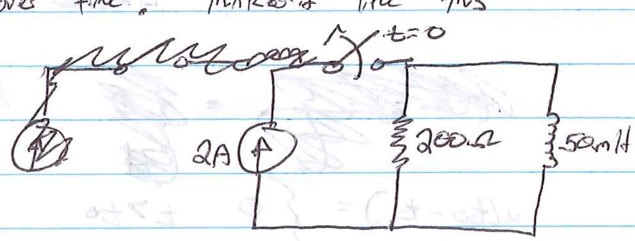
Simple RL circuit.



If initial current of inductor is 2A, at t=0

$$i_L = I_0 e^{-Rt/L}$$

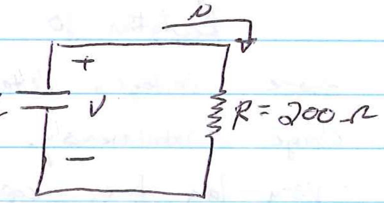
current decays over time. Think of it like this



current source goes away at time = 0. energy leaves the circuit through dissipation in the resistor

$$\tau = \frac{L}{R}$$

Simple RC circuit



if initial voltage across capacitor = 2V at t=0

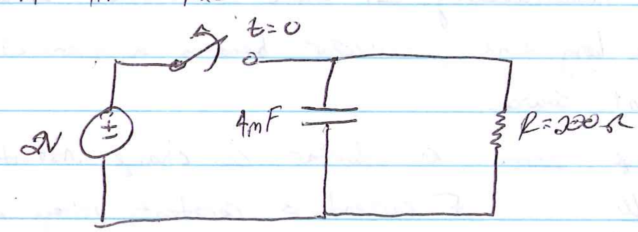
$\tau = RC$

$V(t) = V_0 e^{-t/\tau}$

$V(t) = 2 e^{-t/RC}$

Voltage potential across capacitor ~~&~~ decays over time
The capacitor discharges.

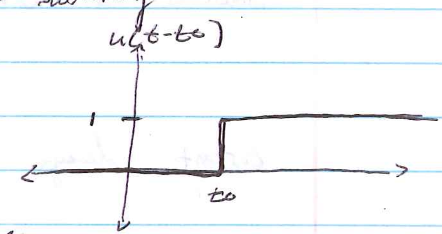
Think of it like this:



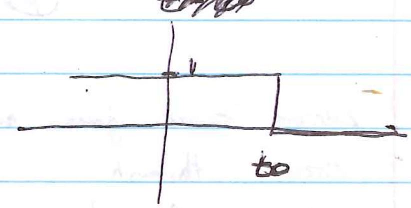
Voltage source goes away at t=0 energy leaves circuit through resistor after ~~t=0~~ Voltage discharges.

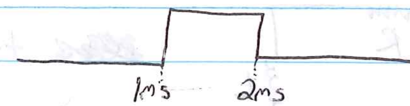
For inductors $i_L(0^-) = i_L(0^+)$ after switching
For capacitors $v_C(0^-) = V_C(0^+)$ after switching

Unit step: $u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$



$u(t_0 - t) = \begin{cases} 0 & t > t_0 \\ 1 & t < t_0 \end{cases}$



$u(t - 1ms) - u(t - 2ms) \quad =$ 

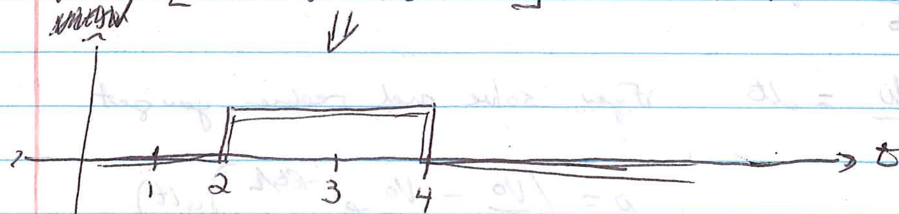
$\sin(2\pi t)$ — ω

$T = \frac{1}{f} = 1s$

$\omega = 2\pi f \qquad f = \frac{\omega}{2\pi} \qquad f = \frac{2\pi}{2\pi} = 1Hz$

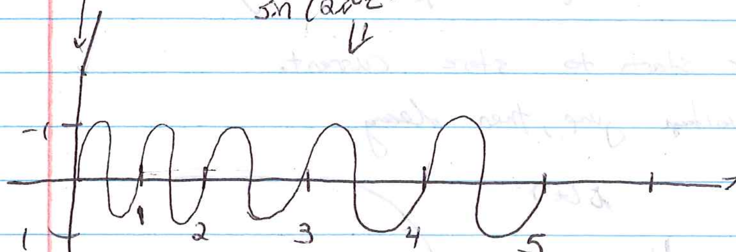
$V(t) = [u(t-2) - u(t-4)] \cdot \sin(2\pi t)\ V$

$\Downarrow$



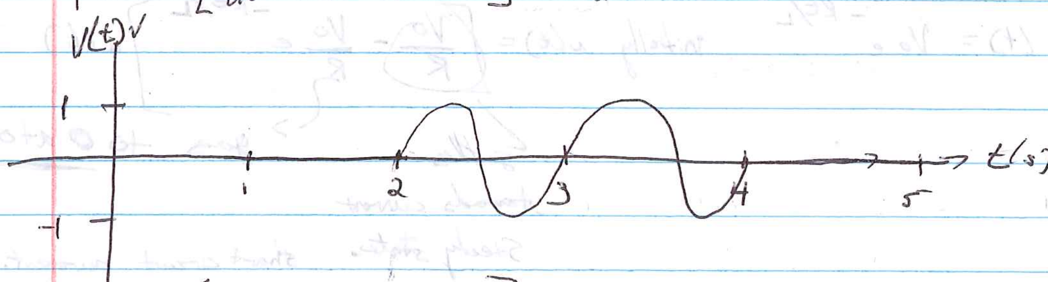
$\sin(2\pi t)$

$\Downarrow$

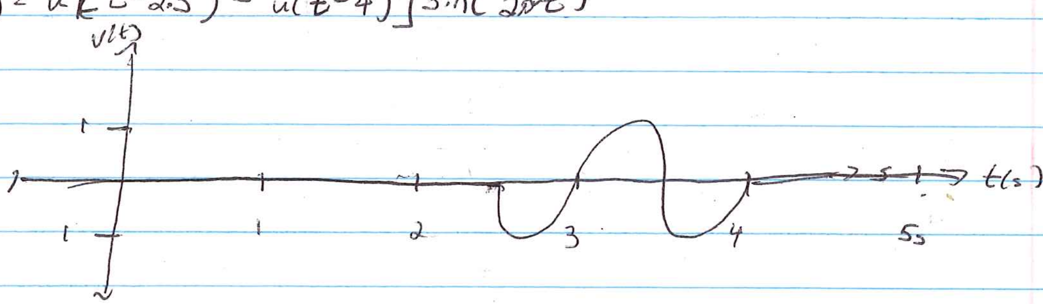


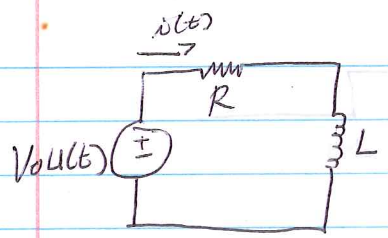
$\Downarrow$

$[u(t-2) - u(t-4)] \cdot \sin(2\pi t)$



$V(t) = u[t-2.5) - u(t-4)] \sin(2\pi t)$





$t<0$, $Ri + L\frac{di}{dt} = 0$

$i = 0$, $\frac{di}{dt} = 0$

$t \geq 0$, $Ri + L\frac{di}{dt} = V_0 u(t)$

$u(t) = 1$ $t \geq 0$ $\quad Ri + L\frac{di}{dt} = V_0$

$$L\frac{di}{dt} = V_0 - Ri$$

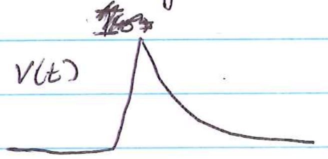
$$\frac{L\,di}{V_0 - Ri} = dt$$ if you solve and reduce, you get

$$i = \left(\frac{V_0}{R} - \frac{V_0}{R}e^{-Rt/L}\right)u(t)$$

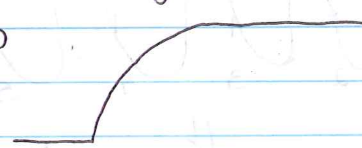
Think about it. Inductor starts to store current.

Initially sharp voltage jump, then decay.

V(t)



i(t)



$\Downarrow$

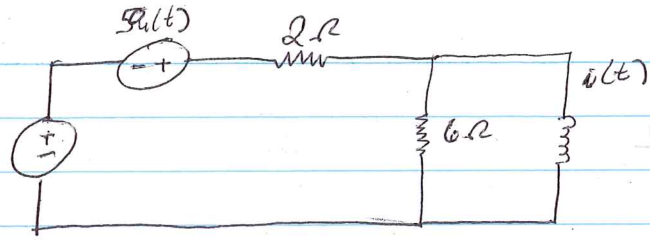
Inverse of each other

$V_L(t) = V_0 e^{-Rt/L}$ $\quad$ initially $i(t) = \left[\frac{V_0}{R} - \frac{V_0}{R}e^{-Rt/L}\right]u(t)$

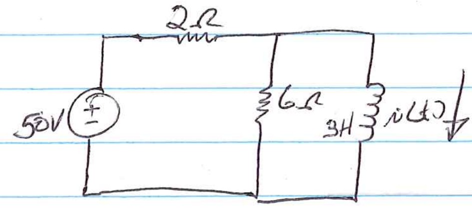
settles towards current steady state. short circuit current.

goes to 0 at ∞

Example 8.8



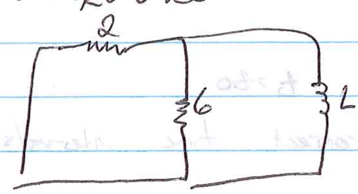
t < 0



$i(t)$ = S.C. current = $\frac{50}{2}$ = 25A

when 50u(t) turns on, $i(t)$ charges to ~~50~~ $\frac{100}{2}$ = 50A.

τ → open inductors



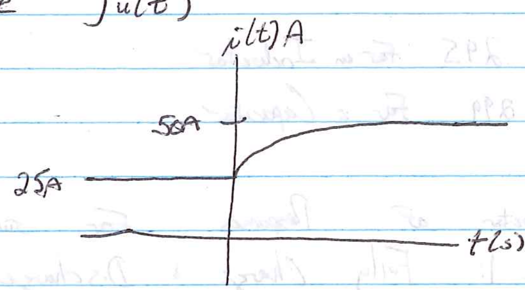
$L = 3H$

$R_{eq} = 2 \| 6 = \frac{12}{8} = \frac{3}{2}$

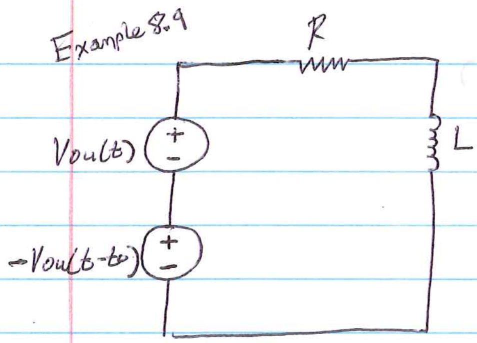
$\therefore \tau = \frac{L}{R} = \frac{3}{3/2} = \frac{1}{2}$

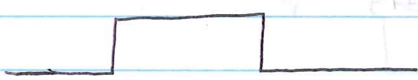
$\therefore i = \frac{50}{2} + \frac{50}{2}\left(1 - e^{-t/\tau}\right)$

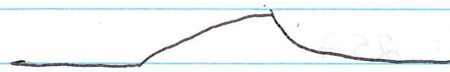
$= 25 + 25\left(1 - e^{-t/2}\right)u(t)$



Example 8.9







$i_1(t)$ when pulse is on $= \frac{V_0}{R}\left(1-e^{-t/\tau}\right)$

$= \frac{V_0}{R} - \frac{V_0}{R}e^{-t/\tau}$ $\quad t>0 \quad i_1=0$ at $t=0$

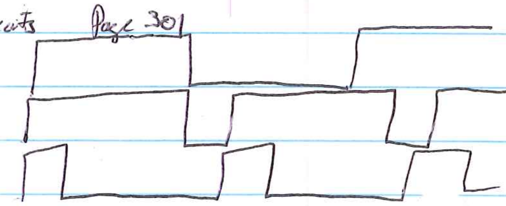
other source : $= \frac{-V_0}{R}\left(1-e^{-(t-t_0)/\tau}\right)$ $\quad t>t_0$

can add solutions together over correct time intervals

$$i(t) = \begin{cases} \frac{V_0}{R}\left(1-e^{-Rt/L}\right) & 0<t<t_0 \\ \frac{V_0}{R}\left(1-e^{-Rt/L}\right) - \frac{V_0}{R}\left(1-e^{-R(t-t_0)/L}\right) & \end{cases}$$

Page 295 For an Inductor
Page 299 For a Capacitor

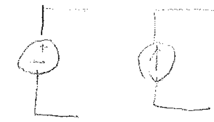
Prediction of Responses For switching circuits Page 301



Case 1: Fully Charge + Discharge
Case 2: Fully charge but not Fully discharge
Case 3: no time to Fully charge, but to discharge
Case 4: no time to Fully charge or charge

Find R_{TH}

<4> Source Transformation



<1> finding V_{oc} & I_{sc} , take their ratio

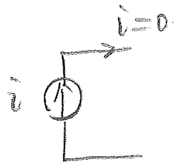
<2> Setting all independent Source to zero , and using resistor combination technique

<3> Connecting an unknown test current source to the terminal , $\boxed{\text{zero out}}$ all other source

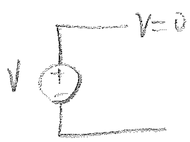
finding an algebraic exppression for voltage / current ,

taking the ratio.

Q: How to zero out or set a source to zero ?



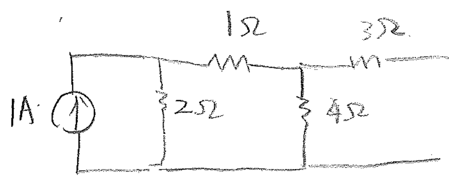
$\Rightarrow$ open circuit



$\Rightarrow$ short circuit

eg. HW 7 Chapter 5. 35

36.



find R_{TH} ?

* when we do superposition $\rightarrow$ zero out

open current source

short voltage source

* Time Constant (unit : s).

$\tau = \frac{L_{eq}}{R_{eq}}$

$\tau = R_{eq} C_{eq}$

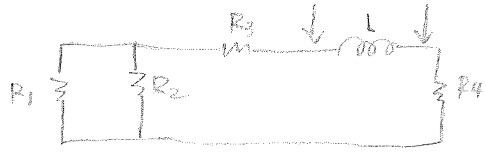
How to find L_{eq} R_{eq} C_{eq}

Ex 8.4 (textbook)

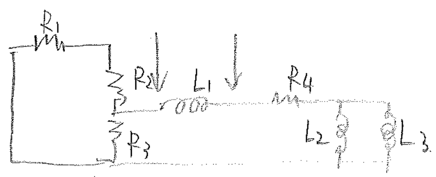
$\boxed{\text{Thevenin equivalent}}$ resistance seen by energy storage element (zero out independent sources)

<1> R_{TH} is the equivalent resistance "seen" by the inductor

Basic Circuit

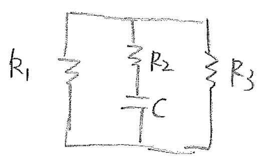


$R_{eq} = R_3 + R_1 // R_2 + R_4$



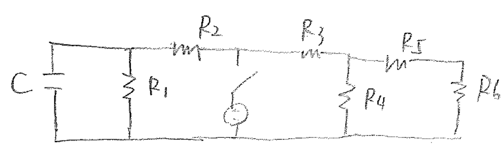
$R_{eq} = (R_1 + R_2) // R_3 + R_4$

$L_{eq} = L_1 + L_2 // L_3$



$R_{eq} = R_2 + R_1 // R_3$

4.8
0.8
5.4

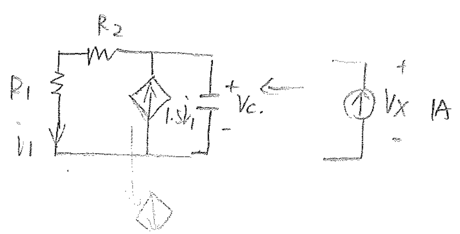


$R_{eq} = \left\{ \left[(R_5 + R_6) // R_4 \right] + R_2 + R_3 \right\} // R_1$

<2> Complicated Circuit (with dependent source)

Thevenin

Connecting a test source to determine the equivalent



$V_X = (1.5 i_1 + 1)(R_1 + R_2)$

$i_1 = \frac{V_X}{R_1 + R_2}$

$\leftarrow R_1 = 20\Omega$

$R_2 = 15\Omega$

$V_X = -60V$ $\quad R_{eq} = -60\Omega$ $\quad \tau = -60\mu s$

exponentially increase