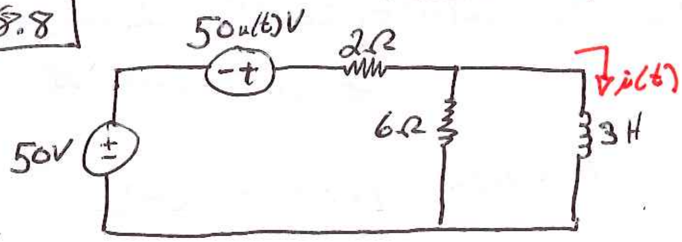
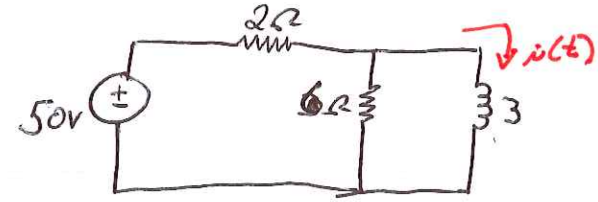
4/7/2015

Example 8.8

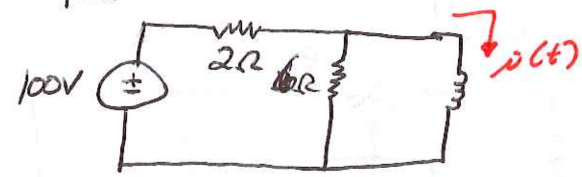


t<0



$i(t<0) = \frac{50}{2} = 25A$

t=∞

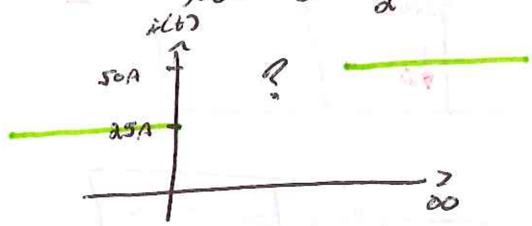


when $t \geq 0$ and $50u(t)$ turns on, we have:

$\tau = \frac{L}{R_{eq}} = \frac{3}{3/2} = 2s \rightarrow$ settling time ~ 4τ to 5τ

$R_{eq} = \frac{2 \times 6}{2+6} = \frac{12}{8} = 1.5$

$i(t=\infty) = \frac{100}{2} = 50A$



Forced Response is a constant current from DC source

In general $i = i_f + i_n$, in our case i_n is a negative exponential

$i_n = Ke^{-t/\tau} = Ke^{-t/2}$

i_f = Forced = 50A $\quad \therefore \; i = 50 + Ke^{-t/2}$ A for $t > 0$

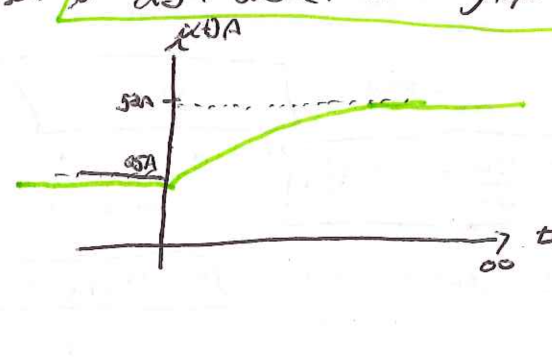
we know prior info and fact that i can't change instantaneously

$\therefore \; 25 = 50 + Ke^{-t/\tau} \Rightarrow 25 = 50 + K \quad \therefore \boxed{K = -25}$

$\therefore \; i = 50 - 25e^{-t/2}$ A for $t > 0$

$i = 25$ A $t < 0$

using a unit step for complete response: $\boxed{i = 25 + 25(1 - e^{-t/2})u(t) \text{ A}}$ for all t



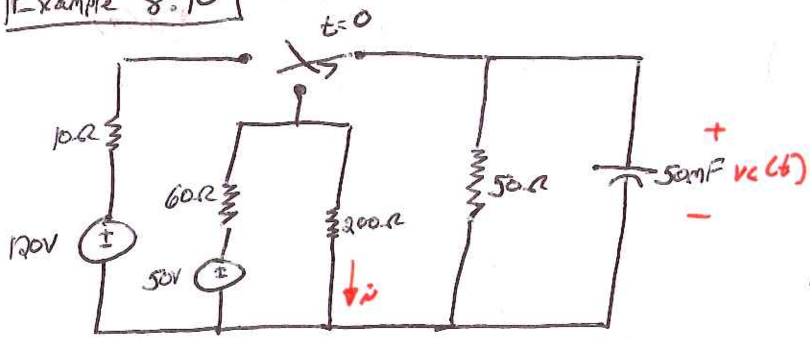
In the most general form, remember: $i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$

$☆\ i_L(0^-) = i_L(0^+)\ ☆$

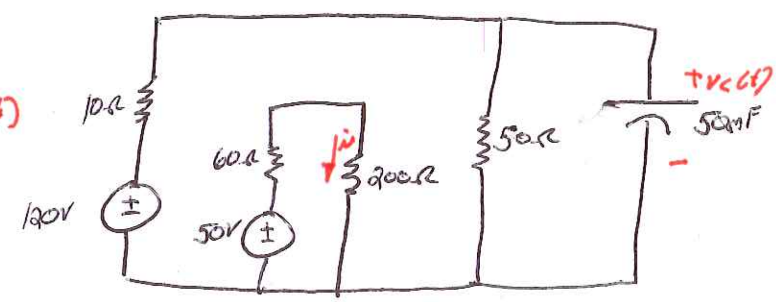
$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$☆\ v_c(0^-) = v_c(0^+)$

Example 8.10

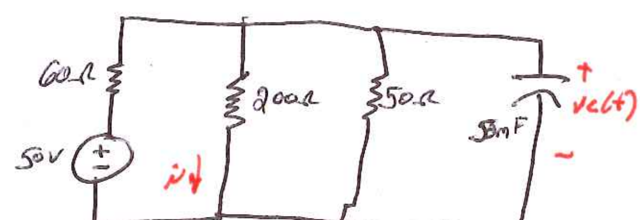


$t \le \phi$



$v_c(0^-) = \frac{50}{10+50} \times 120 = \boxed{100V = v_c(0^-)}$

$t > \phi$



$R_{eq} = 60 \| 200 \| 50$

$40 \| 60 = 24Ω = R_{eq}$

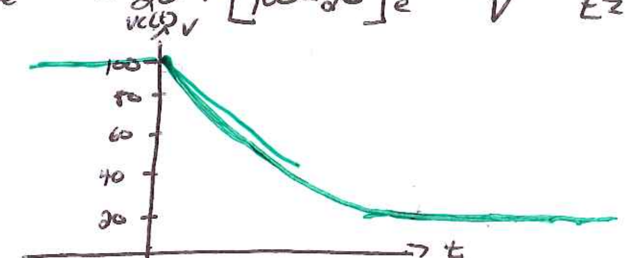
$\tau = R_{eq}C = 24 \times 50 \times 10^{-3} = 1.2$

$200 \| 50 = 40Ω$

$v_c(\infty) = \frac{40}{100} \times 50 = \boxed{20V = v_c(\infty)}$

$\therefore v_c(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = 20 + [100-20]e^{-t/1.2}\ V \quad t \ge \phi$

$\therefore \boxed{v_c(t) = 20 + 80e^{-t/1.2}u(t)\ V}$



$i_F = \frac{20V}{200Ω} = \boxed{0.1A = i_F = i(\infty)}$

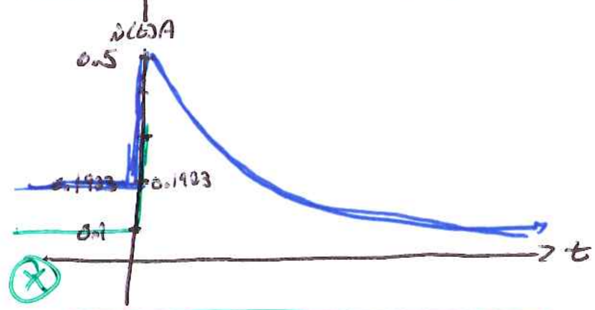
$i_N = Ae^{-t/1.2}$

$\boxed{i = 0.1 + Ae^{-t/1.2}}$

$\boxed{i(0^+) = \frac{100}{200} = \boxed{0.5 Amps}}$

$\hookrightarrow 0.5 = 0.1 + A$

$\therefore A = 0.4 Amps$



also $i(0^-) = \frac{50V}{200+60} = \boxed{0.1923 Amps = i(0^-)}$

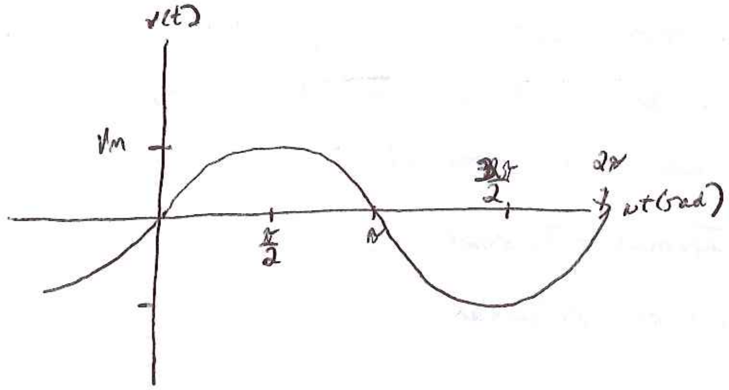
$$\boxed{\begin{aligned} i(t) &= 0.1923u(-t) + [0.1 + 0.4e^{-t/1.2}]u(t) \\ &= 0.1923u(-t) + \{0.1 + [0.5 - 0.1]e^{-t/1.2}\}u(t) \end{aligned}}$$

Chapter 10 — Sinusoids + steady state analysis.

Sinusoidally varying voltage: $v(t) = V_m \sin \omega t$

V_m = Amplitude
ωt = argument w/ t = time +
ω = radians or argument Frequency

Can be plotted as a function of ωt or t



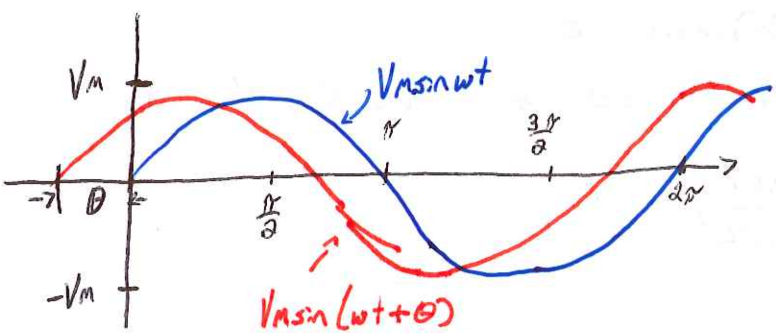
Function repeats every 2π radians w/ $\boxed{\text{period } 2\pi \text{ radians!}}$

when plotted as a function of t, periodic T, such that a sine wave of period T, repeats $\frac{1}{T}$ periods per second which is its Frequency $\frac{1}{T}$ Hertz

$f = \frac{1}{T}$, $\therefore$ $\omega T = 2\pi$

such that $\omega = \frac{2\pi}{T}$ or $= 2\pi f$

More general form with phase θ $\quad v(t) = V_m \sin(\omega t + \theta)$



θ represents # of radians the original sine wave is shifted left or right in time.

~~Vmsinwt~~ $V_m \sin(\omega t + \theta)$ occurs θ rads or θ/ω seconds earlier than $V_m \sin \omega t$ therefore it leads by θ

$\sin \omega t$ lags $\sin(\omega t + \theta)$ by θ rad or leads by $-\theta$ rad

Phase is usually represented in degrees rather than radians

$$v = 100 \sin\left(2\pi 1000 t - \frac{\pi}{6}\right) \Rightarrow v = 100 \sin(2\pi 1000 t - 30^\circ)$$

$\frac{\pi}{6} = 30^\circ$

$\frac{\pi}{4} = 45^\circ$

$\frac{\pi}{3} = 60^\circ$

$\frac{\pi}{2} = 90^\circ$

sines and cosines are 90° out of phase

$\sin \omega t = \cos(\omega t - 90^\circ)$

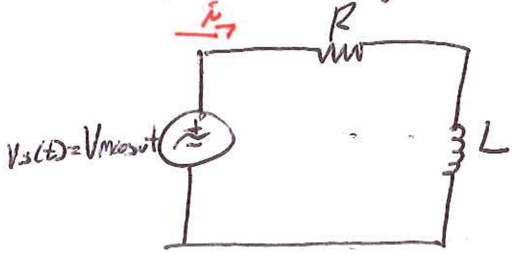
cosine leads sine

Rules for comparing sine waves:

1. Both functions written as sines or cosines
2. Both written with positive amplitudes
3. Each have the same Frequency

Steady State Responses

w/ DC forcing functions, remain same w/ time, but what about sinusoidal sources?



$$V_m \cos \omega t = L\frac{di}{dt} + Ri$$

if at any time the derivative is ϕ,

we have i of the form $\cos\omega t$

But when current $= \phi$, we have the derivative proportional to $\cos\omega t$, implying a current of $\sin\omega t$ $\therefore$ the forced

function may look like: $i(t) = I_1 \cos\omega t + I_2 \sin\omega t$

where I_1 + I_2 are real values that depend on $V_m, R, L + \omega$.

Substitute form, we have $V_m\cos\omega t = $ ~~-...~~

$$L\left[-I_1\omega\sin\omega t + I_2\omega\cos\omega t\right] + R\left[I_1\cos\omega t + I_2\sin\omega t\right]$$

rearrange + regroup

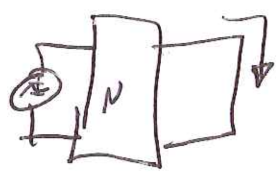
$$= (-LI_1\omega + RI_2)\sin\omega t + (LI_2\omega + RI_1 - V_m)\cos\omega t = \phi$$

set $t = 0 + 1$ to solve for $-LI_1\omega + RI_2 = 0$ + $LI_2\omega + RI_1 - V_m = 0$

$$\therefore \quad I_1 = \frac{RV_m}{R^2 + \omega^2 L^2} \qquad I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

Identities $\quad e^{j\theta} = \cos\theta + j\sin\theta$

Complex forcing function : $V_m\cos(\omega t + \theta)$



$I_m\cos(\omega t + \phi)$

Forcing function produces steady state current

$j = \sqrt{-1} \Rightarrow$ Imaginary sources lead to imaginary responses

$jV_m\sin(\omega t + \theta)$

$jI_m\sin(\omega t + \phi)$

Therefore a complex source creates a complex response

$$V_m \cos(\omega t+\theta) + jV_m \sin(\omega t+\theta) \Rightarrow I_m \cos(\omega t+\phi) + jI_m \sin(\omega t+\phi)$$

☆ $\cos(\omega t+\theta) + j\sin(\omega t+\theta) = e^{j(\omega t+\theta)}$ ☆

$\therefore V_m e^{j(\omega t+\theta)}$ produces $I_m e^{j(\omega t+\phi)}$

Phasors: $e^{jx} = \cos x + j\sin x$ $3V\,dc = ?$ $3e^{j0^\circ} = 3V$

Real 3, 0 imaginary

if $v(t) = V_m \cos \omega t \rightarrow V_m \cos(\omega t + 0^\circ) \rightarrow V_m \angle 0^\circ$ polar complex form

Same notation for current

if $i(t) = I_m \cos(\omega t+\phi)$; $i(t) = Re\{I_m e^{j(\omega t+\phi)}\}$

$$\mathbb{I} = I_m e^{j\phi}$$

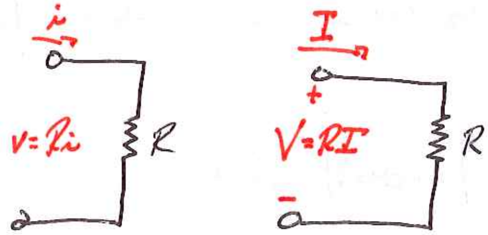
In polar form $\mathbb{I} = I_m \angle\phi$

Phasor transformation: $i(t) = I_m \cos(\omega t+\phi)$ (1) $\rightarrow i(t) = Re\{I_m e^{j(\omega t+\phi)}\}$ (2)

(3) $\mathbb{I} = I_m e^{j\phi} \rightarrow$ (4) $\mathbb{I} = I_m\angle\phi$ ω is known.

Example 10.3

Resistor



$v(t) = Ri(t)$

if $v(t) = V_m e^{j(\omega t+\theta)} = V_m \cos(\omega t+\theta) + jV_m \sin(\omega t+\theta)$

resulting $i(t) = I_m e^{j(\omega t+\phi)} = I_m \cos(\omega t+\phi) + jI_m \sin(\omega t+\phi)$

$\therefore V_m e^{j(\omega t+\theta)} = Ri(t) = I_m e^{j(\omega t+\phi)}$

~~in Polar Form~~ Removing or dividing by $e^{j\omega t}$

$$V_m e^{j\theta} = R I_m e^{j\phi}$$

in polar form $V_m\angle\theta = R I_m \angle\phi$

$\therefore V = RI$

if $V = 8\cos(100t - 50^\circ)V$ if $R = 4\Omega$

$I = \frac{v(t)}{R} = 2\cos(100t - 50^\circ)$ A

$\cancel{i(t) = \frac{v(t)}{R}} =$ $\frac{8\angle -50^\circ}{4} = ?$

in phasor form $\rightarrow$ $2\angle -50^\circ$ A

much easier

Inductor $v(t) = L\frac{di}{dt}$

$$V_m e^{j(\omega t+\theta)} = L\, I_m e^{j(\omega t+\phi)} \frac{d}{dt}$$

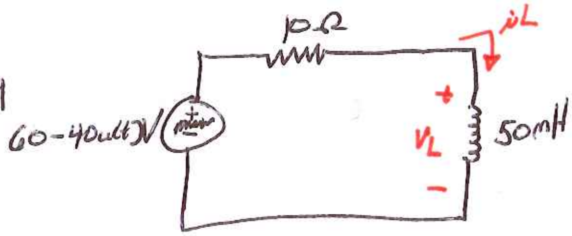
$$V_m e^{j(\omega t+\theta)} = j\omega L\, I_m e^{j(\omega t+\phi)}$$

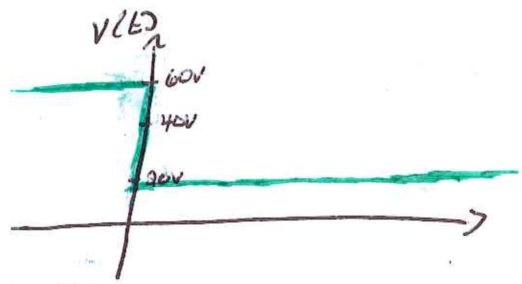
divide by $e^{j\omega t}$ $\quad V_m e^{j\theta} = j\omega L\, I_m e^{j\phi}$

Phasor relationship : $\boxed{\mathbf{V} = j\omega L \mathbf{I}}$

Review

Practice 8.9





$i_L(0^-) = \frac{60}{10} = 6A$ $\qquad \tau = \frac{L}{R} = \frac{50\times10^{-3}}{10} = 50\times10^{-4}$

$i_L(0^+) = i_L(0^-) = 6A$

$i_L(\infty) = \frac{20}{10} = 2A$ $\qquad i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$

$\qquad = 2 + 4e^{-200t}$ for $t>0$

$i_L(3ms) = 4.2A$

$V_L(0^-) = 0V$

$V_L(\infty) = 0V$

$V_L(0^+) =$ ~~[illegible]~~ ~~[illegible]~~ $= 20 - 10(6) =$ voltage drop across inductor $= -40V$ $\quad |V_L(0^+)| = 40V$

$V_L(3m) = 20 - 4.2\times10 = -21.95V \quad |V_L(3ms)| = 21.95V$